AD-A159 885   A LOCAL REFINEMENT FINITE ELEMENT METHOD FOR TIME DEPENDENT PARTIAL DIFFE..(U) ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER WATERVLIET NY L..   1/1

UNCLASSIFIED   J E FLAHERTY ET AL.   AUG 85 ARLCB-TR-85028   F/G 12/1   NL

END
FILMED
DTIC

E440300

(2)

AD-A159 885

AD

TECHNICAL REPORT ARLCB-TR-85028

A LOCAL REFINEMENT FINITE ELEMENT METHOD FOR TIME DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS

JOSEPH E. FLAHERTY

PETER K. MOORE

AUGUST 1985

U.S. ARMY ARDC

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER

LARGE CALIBER WEAPON SYSTEMS LABORATORY

BENÉT WEAPONS LABORATORY

WATERVLIET N.Y. 12189

85 09 23 026

DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

The use of trade name(s) and/or manufacture(s) does not constitute an official indorsement or approval.

DISPOSITION

Destroy this report when it is no longer needed. Do not return it to the originator.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARLCB-TR-85028	2. GOVT ACCESSION NO. AD-A159885	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A LOCAL REFINEMENT FINITE ELEMENT METHOD FOR TIME DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Joseph E. Flaherty and Peter K. Moore (See Reverse)		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Armament Research & Development Center Benet Weapons Laboratory, SMCAR-LCB-TL Watervliet, NY 12189-5000		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCMS NO.6111.01.91A0.011 PRON NO. 1A425M51A1A
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Research & Development Center Large Caliber Weapon Systems Laboratory Dover, NJ 07801-5001		12. REPORT DATE August 1985
		13. NUMBER OF PAGES 17
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES Presented at the Second Conference on Applied Math and Computing, 22-25 May 1984, RPI, Troy, New York.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Finite Element Method
Partial Differential Equations
Space-Time Elements
Local Refinement
Adaptive Methods

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We discuss an adaptive local refinement finite element method for solving initial-boundary value problems for vector systems of partial differential equations in one space dimension and time. The method uses piecewise bilinear rectangular space-time finite elements. For each time step, grids are automatically added to regions where the local discretization error is estimated as being larger than a prescribed tolerance. We discuss several

(CONT'D ON REVERSE)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

7. AUTHORS (CONT'D)

Joseph E. Flaherty
Armament Research and Development Center
Large Caliber Weapon Systems Laboratory
Watervliet, NY 12189-5000
and
Department of Computer Science
Rensselaer Polytechnic Institute
Troy, NY 12181

Peter K. Moore
Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, NY 12181

20. ABSTRACT (CONT'D)

aspects of our algorithm, including the tree structure that is used to represent the finite element solution and grids, an error estimation technique, and initial and boundary conditions at coarse-fine mesh interfaces. We also present computational results for a simple linear hyperbolic problem, a problem involving Burgers' equation, and a model combustion problem.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

INTRODUCTION

There is an ever increasing need to solve problems of greater complexity and a corresponding need for reliable and robust software tools to accurately and efficiently describe the phenomena. Adaptive techniques are good candidates for providing the computational methods and codes necessary to solve some of these difficult problems. Two popular adaptive techniques are: (1) moving mesh methods, where a grid of a fixed number of finite difference cells or finite elements is moved in order to follow and resolve local nonuniformities in the solution, and (2) local refinement methods, where uniform fine grids are added to coarser grids in regions where the solution is not adequately resolved. A representative sample of both types of methods is contained in Babuska, Chandra, and Flaherty (ref 1). Recently, Adjerid and Flaherty (ref 2) developed a finite element method that combines mesh moving and refinement.

Herein, we discuss a local refinement finite element procedure for finding numerical solutions of M-dimensional vector systems of partial differential equations having the form

$$Lu := u_t + f(x,t,u,u_x) - [D(x,t,u,u_x]_x = 0 \quad , \quad a < x < b, \; t > 0 \qquad (1)$$

subject to the initial conditions

$$u(x,0) = u^0(x) \quad , \quad a \leq x \leq b \qquad (2)$$

and appropriate boundary conditions so that the problem has a well-posed solution.

---

[1]Babuska, I., Chandra, J., and Flaherty, J. E. (eds.), Adaptive Computational Methods for Partial Differential Equations, SIAM, Philadelphia, 1983.
[2]Adjerid, S. and Flaherty, J. E., "A Moving Finite Element Method for Time Dependent Partial Differential Equations with Error Estimation and Refinement," to appear in SIAM J. Numer. Anal., 1985.

We discretize Eqs. (1) and (2) for a time step using a finite element-Galerkin procedure with piecewise bilinear approximations on a rectangular space-time net. At the end of each time step, we estimate the local discretization error, add finer subgrids of space-time elements in regions of high error, and recursively solve the problem again in these regions. The process terminates when the error estimate on each grid is less than a prescribed tolerance. The original coarse space-time grid is then carried forward for the next time step and the strategy is repeated. Our algorithm is discussed further in Flaherty and Moore (ref 3) and some of this discussion is repeated in the following section.

Berger (ref 4) used a similar local refinement procedure to solve one- and two-dimensional hyperbolic systems. She used explicit finite difference schemes to discretize the partial differential equations, while we use implicit finite element techniques since we are primarily interested in parabolic problems.

In addition to the discretization technique, the major numerical questions that must be answered as part of the development of a local refinement code are (1) the estimation of the discretization error, and (2) the appropriate initial and boundary conditions to apply at coarse-fine mesh interfaces. Of course computer science questions, such as which language to

---

[3]Flaherty, J. E. and Moore, P. K., "An Adaptive Local Refinement Finite Element Method for Parabolic Partial Differential Equations," Proceedings of the International Conference on Accuracy and Estimates and Adaptive Refinements in Finite Element Computations, Technical University of Lisbon, Lisbon, Vol. 2, 1984, pp. 139-152.

[4]Berger, M. J., "Adaptive Mesh Refinement for Hyperbolic Partial Differential Equations," Report No. STAN-CS-82-924, Department of Computer Science, Stanford University, 1982.

use to describe and implement the various algorithms and what data structures to use to represent and store the grids and solutions, must also be answered. Our work in all of these areas is still far from complete, and herein we only discuss our progress and thoughts on error estimation techniques, data structures, and interface conditions (see the following section). We also present the results of three examples that illustrate our method and some preliminary conclusions and future plans.

FINITE ELEMENT ALGORITHM

We discretize Eq. (1) on a strip $\alpha < x < \beta$, $p < t < q$ using a finite element-Galerkin method with a uniform grid of N rectangular elements of size $(\beta-\alpha)/N$ by $(q-p)$. We refer to this grid as $R(\alpha,\beta,p,q,N,f,s)$, where f and s are pointers to the father and son grids discussed later. Each grid uses records to store the appropriate information.

We generate the discrete system on $R(\alpha,\beta,p,q,N,f,s)$ in the usual manner; thus, we approximate u by $U(x,t)$ and select test functions $V(x,t)$, where U and V are elements of a space of C^0 bilinear polynomials with respect to the grid R. We then take the inner produce of Eq. (1) and V, replace u by U, and integrate any diffusive terms by parts to obtain

$$\int_R [V^T U_t + V^T f(x,t,U,U_x) + V_x^T D(x,t,U)U_x]dxdt$$

$$- \int_p^q V^T D(x,t,U)U_x \Big|_\alpha^\beta dt = 0 \qquad (3)$$

Equation (3) must vanish for all bilinear functions V on the grid R. The integrals are approximated using a four-point Gauss quadrature rule and the resulting nonlinear system is solved by Newton iteration (see, for example

Reference 5 for additional details). Appropriate initial and boundary conditions for Eq. (3) are discussed later in this section.

We describe our local refinement procedure for solving Eqs. (1) and (2) for one time step (t^0,t^1) on a coarse grid with N^0 elements, i.e., on $R(a,b,t^0,t^1,N^0,0,s)$ (where the pointer f = 0 signifies that this grid has no father). To solve this problem, we simply call the procedure "locref" with the arguments $R(a,b,t^0,t^1,N^0,0,s)$, tol, tsub for each coarse grid time interval. A pseudo-PASCAL description of the procedure "locref" is shown in Figure 1.

```
PROCEDURE locref (R(α,β,p,q,N,f,s), tol, tsub)
   BEGIN
      Solve the finite element equations (Eq. (3)) on R(α,β,p,q,N,f,s);
      Estimate the error on R(α,β,p,q,N,f,s);
      IF error > tol, THEN
         BEGIN
            calculate where error > tol and return the son grids;
            FOR j := 1 TO tsub DO
               FOR i := 1 TO number of sons DO
                  BEGIN
                     p[j] := p + (j-1)*(q-p)/tsub;
                     q[j] := p[j] + (q-p)/tsub;
                     locref (R(α[i],β[i],p[j],q[j],N[i],
                        R(α,β,p,q,N,f,s),s[i],tol,tsub)
                  END
         END
END;
```

Figure 1. Algorithm for local refinement solution of Eqs. (1) and (2) on $R(\alpha,\beta,p,q,N,f,s)$ with an error tolerance of tol and dividing the local time step by tsub each time the error test is not satisfied.

The recursive algorithm locref sets up a tree structure of grids with $R(a,b,t^0,t^1,N^0,0,s)$ being the root node and with the solution being generated

[5] Davis, S. F. and Flaherty, J. E., "An Adaptive Finite Element for Initial-Boundary Value Problems for Partial Differential Equations," SIAM J. Sci. Stat. Comput., Vol. 3, 1982, pp. 6-27.

by a preorder traversal of the tree at each local time step. For example, if the root grid is refined to given two subgrids and the time step is halved, the problem is solved on the first subgrid on its first time step, followed by the second subgrid on the same time step. This procedure is repeated for the second time step. The error is estimated by Richardson extrapolation, i.e., the space and time steps are halved and the problem is solved again on this new grid. The two solutions that are obtained at each original grid point are used to generate an error estimate. If this pointwise estimate exceeds the tolerance "tol", finer grids are added as leaf nodes to the tree. This procedure is similar to one used by Berger (ref 4); however, there are more economical error estimation strategies (see, for example, Bieterman and Babuska (refs 6,7)) which we are currently investigating.

In order to solve the finite element system, Eq. (3), we need to supply initial and boundary conditions. On any grid with $p = 0$, $\alpha = a$, or $\beta = b$, these can be obtained from the initial condition, Eq. (2), or prescribed boundary conditions. However, artificial initial and boundary conditions must be created at all other coarse-fine mesh interfaces. This is a difficult and crucial problem that is discussed for explicit finite difference methods by

---

[4]Berger, M. J., "Adaptive Mesh Refinement for Hyperbolic Partial Differential Equations," Report No. STAN-CS-82-924, Department of Computer Science, Stanford University, 1982.

[6]Bieterman, M. and Babuska, I., "The Finite Element Method for Parabolic Equations, I. A Posteriori Estimation," Numer. Math., Vol. 40, 1982, pp. 339-371.

[7]Bieterman, M. and Babuska, I., "The Finite Element Method for Parabolic Equations, II. A Posteriori Error Estimation and Adaptive Approach," Numer. Math., Vol. 40, 1982, pp. 373-406.

Berger (refs 4,8); however, it is largely unanswered for finite element applications. Instabilities or incorrect solutions (see Example 1 in the following section) can result if inappropriate conditions are specified.

For initial conditions, two strategies immediately come to mind: (1) saving all fine grid data for propagation in time, or (2) interpolating the best coarse grid data to finer grids. We consider a blend of the two strategies which consists of saving the fine grid data down to a given level λ in the tree and subsequently interpolating for finer grids. Each grid in the first λ levels either has a linked list of the initial data directly associated with it or uses an initial data list of an ancestor grid. To find the value of the solution at some new initial point, the coordinate of that point is sequentially compared to values in the linked list until an interval containing the point is found so that interpolation can be used. This is costly and we are investigating more efficient procedures that use the natural ordering that already exist. We used either piecewise linear interpolation or piecewise parabolic interpolation with shape preserving splines developed by McLaughlin (ref 9). For each grid in the first λ levels of the tree, a linked list is created to store the initial data. We are studying several alternative ways of determining a proper value for λ.

---

[4]Berger, M. J., "Adaptive Mesh Refinement for Hyperbolic Partial Differential Equations," Report No. STAN-CS-82-924, Department of Computer Science, Stanford University, 1982.

[8]Berger, M. J., "Stability of Interfaces with Mesh Refinement," Report No. 83-42, Institute for Computer Applications in Science and Engineering, NASA Langley Research Center, Hampton, 1983.

[9]McLaughlin, H. W., "Shape Preserving Planar Interpolation: An Algorithm," IEEE Computer Graphics and Applics., Vol. 3, 1983, pp. 58-67.

At the present time, we prescribe internal Dirichlet boundary conditions by linearly interpolating from coarse to finer grids. A buffer zone of two elements is added to each end of regions of high error that do not intersect the boundaries $x = a$ and b. If two buffer zones overlap or are separated from one another by one element, the two grids are joined. Similarly, if the buffer is only one element away from either a or b, that element is added to the grid.

NUMERICAL EXAMPLES

An experimental code based on the algorithms in the previous section has been written in FORTRAN-77. We are testing it on several examples, some of these follow and others are presented in Reference 3. All results were computed in double precision on an IBM 3081D computer.

Example 1

In order to illustrate the importance of adequately resolving initial conditions at each time step, we solve the linear hyperbolic initial value problem

$$u_t + u_x = 0 ,$$

$$u(x,0) = u^0(x) = \begin{cases} (1/2)(\cos(20\ \pi(x-0.45)-1) , & 0.35 < x < 0.75 \\ 0 , & \text{otherwise} \end{cases}$$

We solve this problem for one coarse time step of $\Delta t = 0.05$, 10 elements on $0 < x < 1$, tol $= 0.01$. For small enough times, the exact solution is $u^0(x-t)$.

[3]Flaherty, J. E. and Moore, P. K., "An Adaptive Local Refinement Finite Element Method for Parabolic Partial Differential Equations," Proceedings of the International Conference on Accuracy and Estimates and Adaptive Refinements in Finite Element Computations, Technical University of Lisbon, Lisbon, Vol. 2, 1984, pp. 139-152.

If initial conditions are interpolated from the coarse to the fine grid, the oscillations are missed and an incorrect solution is computed, possibly without a user realizing that there is anything wrong. However, saving initial values for the first eight levels of the tree of grids calculates the correct solution to the prescribed accuracy. The incorrect and correct solutions are shown at $t = 0.05$ in Figure 2.

<u>Example 2</u>

We solve the following problem for Burgers' equation:

$$u_t + uu_x = du_{xx}\ ,\ 0 < x < 1\ ,\ 0 < t < 1$$

$$u(x,0) = \sin \pi x\ ,\ 0 < x < 1$$

$$u(0,t) = u(1,t) = 0\ ,\ t > 0$$

We choose $d = 0.00003$, a coarse grid of 10 elements and $\Delta t = 0.1$, and piecewise parabolic approximations for the initial conditions with $\lambda = 6$. It is well known that the solution of this problem is a "pulse" that steepens as it travels to the right until it forms a shock layer at $x = 1$. After a time of $O(1/d)$ the pulse dissipates and the solution decays to zero. We solve this problem for tol = 0.01 and 0.001 and show the solutions at $t = 0.4$ in Figure 3. The solution with the cruder tolerance is exhibiting some oscillations that are within our bounds. These, however, are not visible when the finer tolerance is used to solve the problem.

<u>Example 3</u>

We solve the model combustion problem

$$u_t + u_x - 2e^u = u_{xx}\ ,\ 0 < x < 1\ ,\ 0 < t < 1$$

$$u(x,0) = 0\ ,\ u(0,t) = 0\ ,\ u_x(1,t) = 0$$

The exponential nonlinearity is typical in combustion problems having Arrhenius chemical kinetics. However, in this case the solution develops a "hot spot" at $x = 1$ and becomes infinite when t is approximately 0.85. We choose a coarse grid of 20 elements and $\Delta t = 0.05$, tol = 0.001, and $\lambda = 6$. In Figure 4 we show the computed solution $U(x,t)$ as a function of x for $t = 0.05$, 0.06, and 0.08, and in Figure 5 we show the mesh that was used to solve the problem. We see that the mesh is initially concentrated in the region near $x = 0$ where the curvature of the solution is largest. As time progresses and the curvature diminishes, excessive refinement is not necessary. Finally, as the solution begins to "blow-up" our algorithm generates a fine mesh only in the region near $x = 1$.

DISCUSSIONS AND CONCLUSIONS

We have briefly described an adaptive local refinement algorithm for solving time dependent partial differential equations. Even though this is very much a working algorithm and not a production code, we are very encouraged by the preliminary results. We are investigating several possible ways of improving the efficiency and robustness of our algorithm. These include adding higher order polynomial finite element approximations, adaptively changing the number of elements that are carried forward in the coarse grid at each coarse time step, selecting the appropriate buffer length, adaptively determining the optimal number of levels of initial conditions at coarse-fine interfaces, and applying the best boundary conditions to apply at internal boundaries. We are encouraged by the performance of McLaughlin's (ref 9) shape preserving parabolic splines; however, the entire area of interpolating from coarse to fine grids needs further study. We are also developing non-Dirichlet "natural" boundary conditions to use at coarse-fine mesh interfaces.

Finally, we are very interested in combining the moving mesh strategy of, for example, References 5 and 10 with the present local refinement strategy and extending our methods to two and three dimensions.

---

[5]Davis, S. F. and Flaherty, J. E., "An Adaptive Finite Element for Initial-Boundary Value Problems for Partial Differential Equations," SIAM J. Sci. Stat. Comput., Vol. 3, 1982, pp. 6-27.

[9]McLaughlin, H. W., "Shape Preserving Planar Interpolation: An Algorithm," IEEE Computer Graphics and Applics., Vol. 3, 1983, pp. 58-67.

[10]Flaherty, J. E., Coyle, J. M., Ludwig, R., and Davis, S. F., "Adaptive Finite Element Methods for Parabolic Partial Differential Equations," in Adaptive Computational Methods for Partial Differential Equations, Babuska, I., Chandra, J., and Flaherty, J. E. (eds.), SIAM, Philadelphia, 1983.

REFERENCES

1. Babuska, I., Chandra, J., and Flaherty, J. E. (eds.), Adaptive Computational Methods for Partial Differential Equations, SIAM, Philadelphia, 1983.

2. Adjerid, S. and Flaherty, J. E., "A Moving Finite Element Method for Time Dependent Partial Differential Equations with Error Estimation and Refinement," to appear in SIAM J. Numer. Anal., 1985.

3. Flaherty, J. E. and Moore, P. K., "An Adaptive Local Refinement Finite Element Method for Parabolic Partial Differential Equations," Proceedings of the International Conference on Accuracy and Estimates and Adaptive Refinements in Finite Element Computations, Technical University of Lisbon, Lisbon, Vol. 2, 1984, pp. 139-152.

4. Berger, M. J., "Adaptive Mesh Refinement for Hyperbolic Partial Differential Equations," Report No. STAN-CS-82-924, Department of Computer Science, Stanford University, 1982.

5. Davis, S. F. and Flaherty, J. E., "An Adaptive Finite Element for Initial-Boundary Value Problems for Partial Differential Equations," SIAM J. Sci. Stat. Comput., Vol. 3, 1982, pp. 6-27.

6. Bieterman, M. and Babuska, I., "The Finite Element Method for Parabolic Equations, I. A Posteriori Estimation," Numer. Math., Vol. 40, 1982, pp. 339-371.

7. Bieterman, M. and Babuska, I., "The Finite Element Method for Parabolic Equations, II. A Posteriori Error Estimation and Adaptive Approach," Numer. Math., Vol. 40, 1982, pp. 373-406.

8. Berger, M. J., "Stability of Interfaces with Mesh Refinement," Report No. 83-42, Institute for Computer Applications in Science and Engineering, NASA Langley Research Center, Hampton, 1983.

9. McLaughlin, H. W., "Shape Preserving Planar Interpolation: An Algorithm," <u>IEEE Computer Graphics and Applics.</u>, Vol. 3, 1983, pp. 58-67.

10. Flaherty, J. E. Coyle, J. M., Ludwig, R., and Davis, S. F., "Adaptive Finite Element Methods for Parabolic Partial Differential Equations," in <u>Adaptive Computational Methods for Partial Differential Equations</u>, Babuska, I., Chandra, J., and Flaherty, J. E. (eds.), SIAM, Philadelphia, 1983.

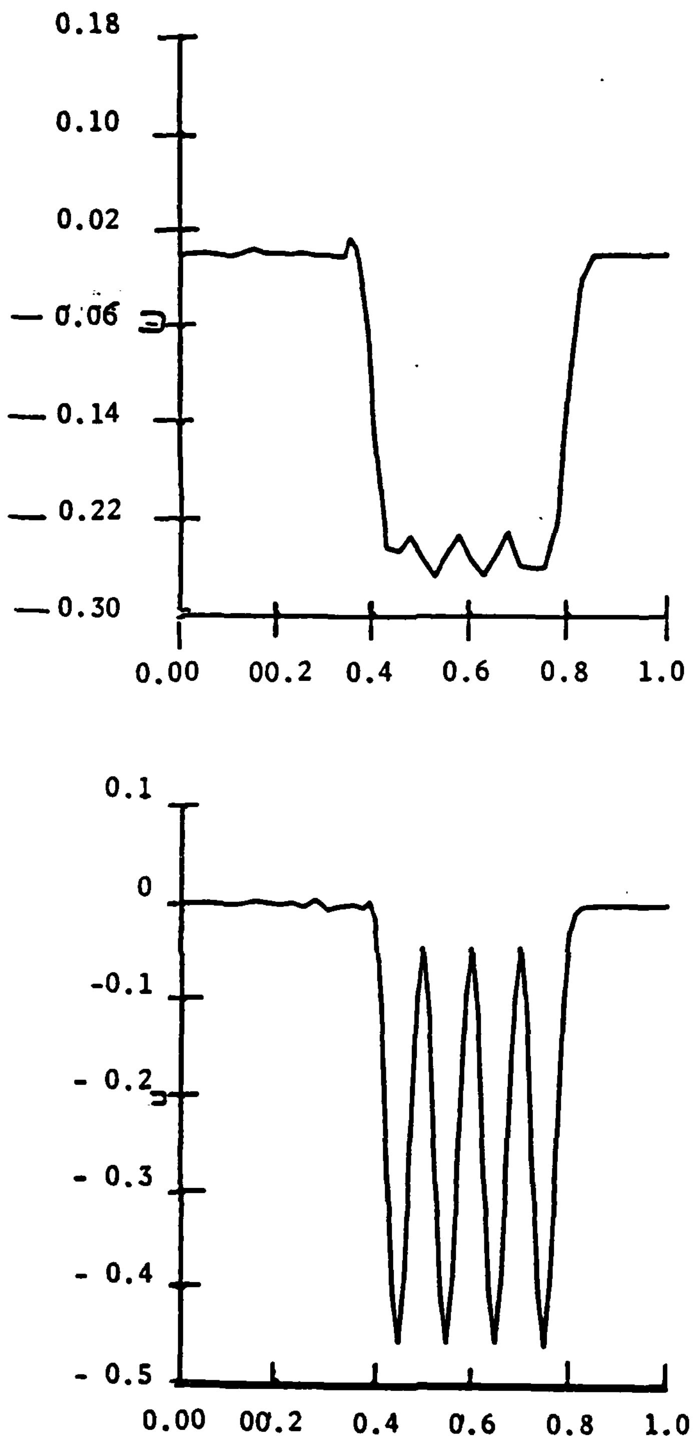
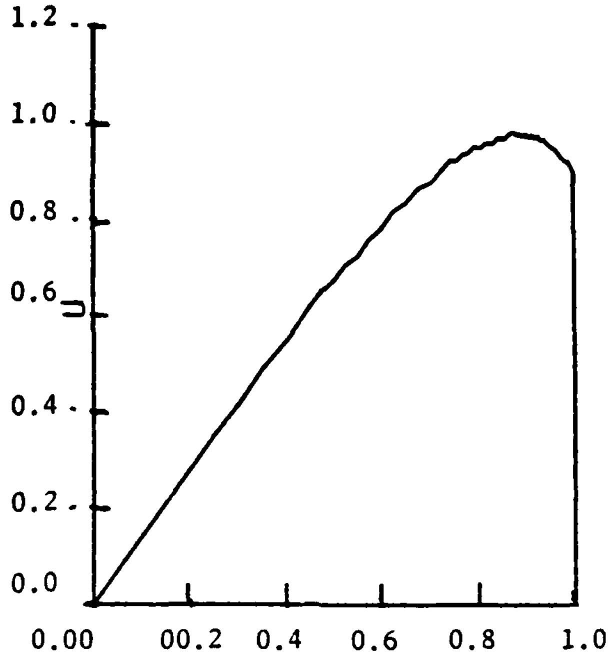


Figure 2. Solution of Example 1 at time t = 0.05 using interpolation from the coarse grid to the fine grid (top) and saving the initial values for the first 8 levels of the tree (bottom). The upper solution overlooks the oscillations and is incorrect.



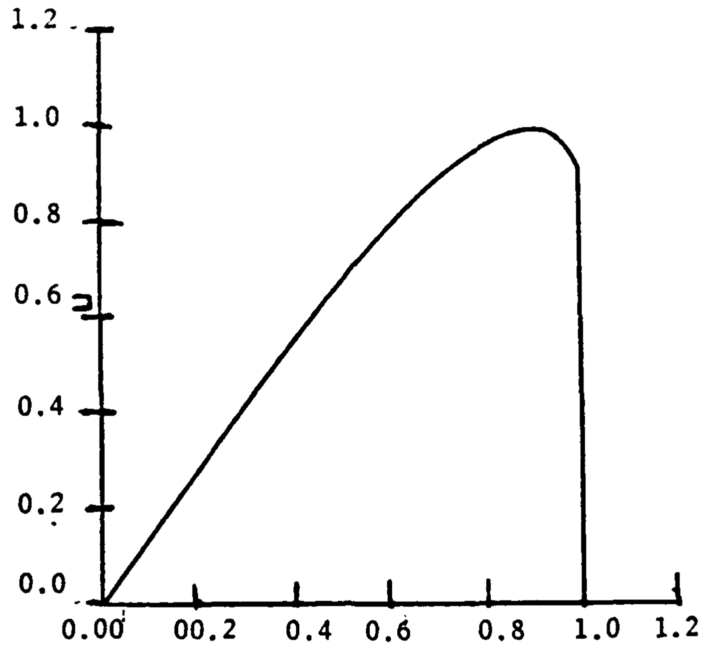


Figure 3. Solution of Example 2 at time t = 0.4 with tolerances of 0.01 (top) and 0.001 (bottom).

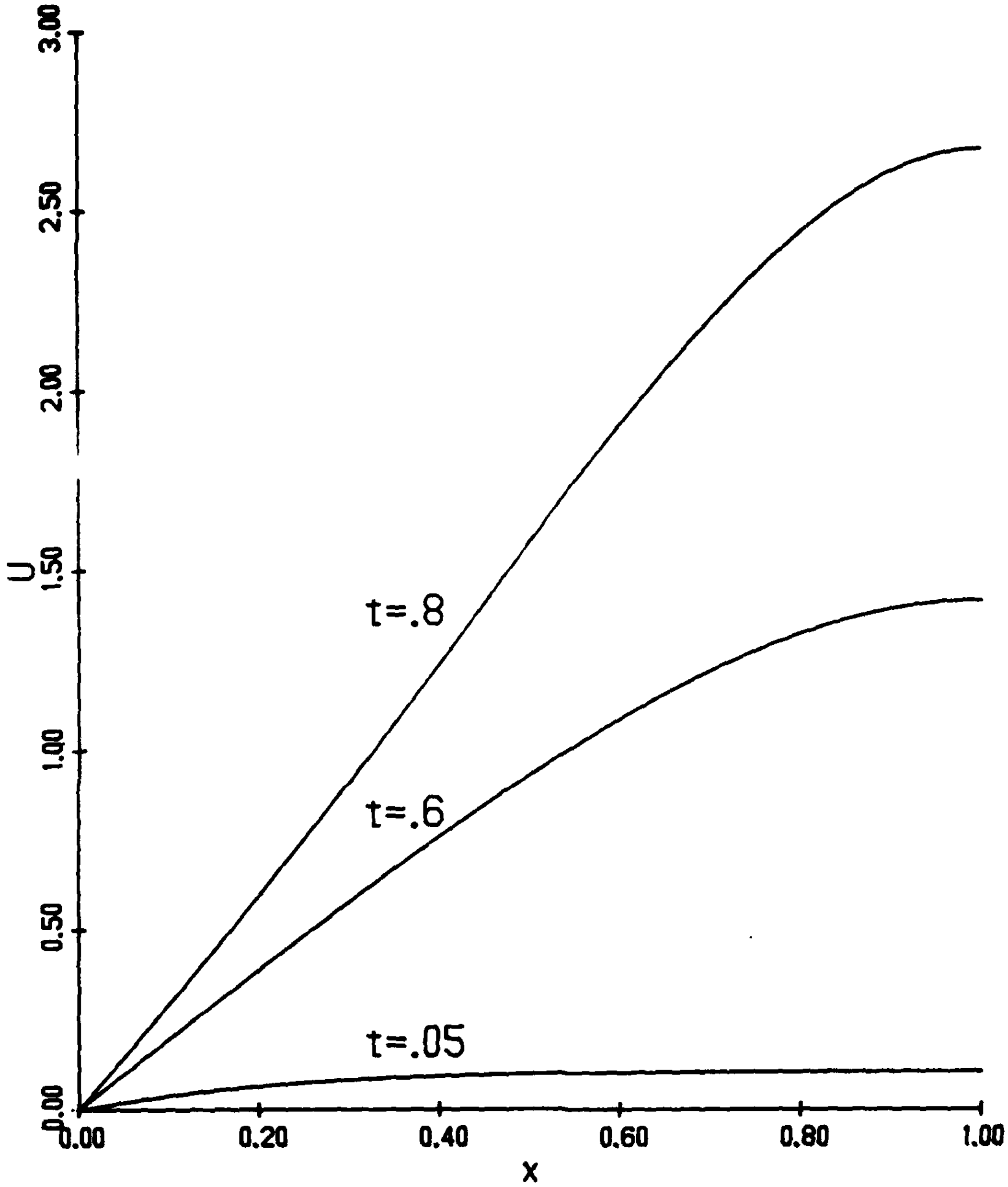


Figure 4. Solution of Example 3 at times t = 0.05, 0.6, and 0.8 with a tolerance of 0.001.

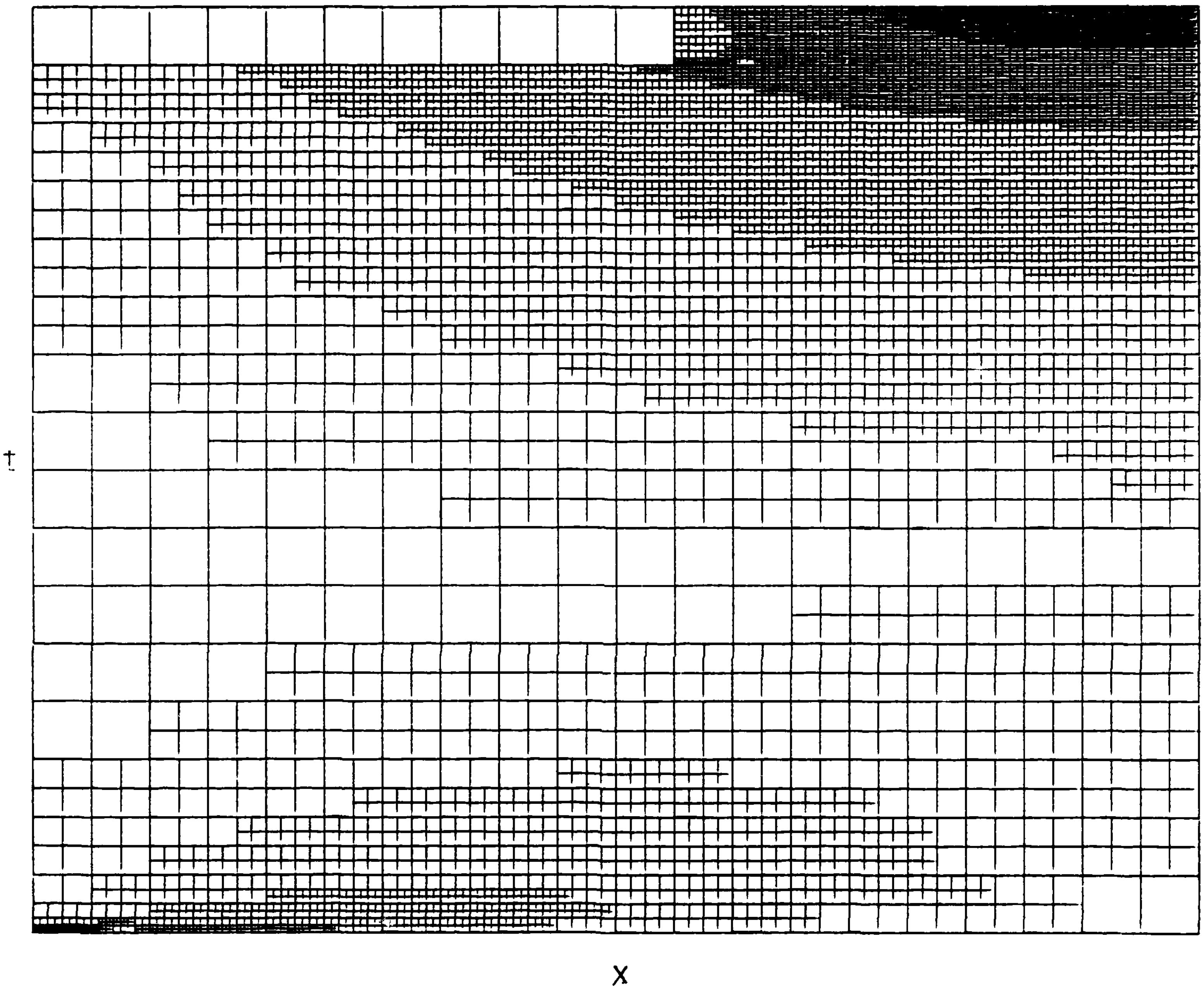


Figure 5. The grids generated in solving Example 3 for $0 < t < 0.8$. The initial coarse mesh has 20 elements with $\Delta t = 0.05$.

TECHNICAL REPORT INTERNAL DISTRIBUTION LIST

	NO. OF COPIES
CHIEF, DEVELOPMENT ENGINEERING BRANCH	
ATTN: SMCAR-LCB-D	1
-DA	1
-DP	1
-DR	1
-DS (SYSTEMS)	1
-DS (ICAS GROUP)	1
-DC	1
CHIEF, ENGINEERING SUPPORT BRANCH	
ATTN: SMCAR-LCB-S	1
-SE	1
CHIEF, RESEARCH BRANCH	
ATTN: SMCAR-LCB-R	2
-R (ELLEN FOGARTY)	1
-RA	1
-RM	2
-RP	1
-RT	1
TECHNICAL LIBRARY	5
ATTN: SMCAR-LCB-TL	
TECHNICAL PUBLICATIONS & EDITING UNIT	2
ATTN: SMCAR-LCB-TL	
DIRECTOR, OPERATIONS DIRECTORATE	1
DIRECTOR, PROCUREMENT DIRECTORATE	1
DIRECTOR, PRODUCT ASSURANCE DIRECTORATE	1

NOTE: PLEASE NOTIFY DIRECTOR, BENET WEAPONS LABORATORY, ATTN: SMCAR-LCB-TL, OF ANY ADDRESS CHANGES.

TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST

	NO. OF COPIES
ASST SEC OF THE ARMY RESEARCH & DEVELOPMENT ATTN: DEP FOR SCI & TECH THE PENTAGON WASHINGTON, D.C. 20315	1
COMMANDER DEFENSE TECHNICAL INFO CENTER ATTN: DTIC-DDA CAMERON STATION ALEXANDRIA, VA 22314	
COMMANDER US ARMY MAT DEV & READ COMD ATTN: DRCDE-SG 5001 EISENHOWER AVE ALEXANDRIA, VA 22333	1
COMMANDER ARMAMENT RES & DEV CTR US ARMY AMCCOM	
ATTN: SMCAR-LC	1
SMCAR-LCE	1
SMCAR-LCM (BLDG 321)	1
SMCAR-LCS	1
SMCAR-LCU	1
SMCAR-LCW	1
SMCAR-SCM-O (PLASTICS TECH EVAL CTR, BLDG. 351N)	1
SMCAR-TSS (STINFO)	2
DOVER, NJ 07801	
DIRECTOR BALLISTICS RESEARCH LABORATORY ATTN: AMXBR-TSB-S (STINFO) ABERDEEN PROVING GROUND, MD 21005	1
MATERIEL SYSTEMS ANALYSIS ACTV ATTN: DRXSY-MP ABERDEEN PROVING GROUND, MD 21005	1

	NO. OF COPIES
COMMANDER US ARMY AMCCOM ATTN: SMCAR-ESP-L ROCK ISLAND, IL 61299	1
COMMANDER ROCK ISLAND ARSENAL ATTN: SMCRI-ENM (MAT SCI DIV) ROCK ISLAND, IL 61299	1
DIRECTOR US ARMY INDUSTRIAL BASE ENG ACTV ATTN: DRXIB-M ROCK ISLAND, IL 61299	1
COMMANDER US ARMY TANK-AUTMV R&D COMD ATTN: TECH LIB - DRSTA-TSL WARREN, MI 48090	1
COMMANDER US ARMY TANK-AUTMV COMD ATTN: DRSTA-RC WARREN, MI 48090	1
COMMANDER US MILITARY ACADEMY ATTN: CHMN, MECH ENGR DEPT WEST POINT, NY 10996	1
US ARMY MISSILE COMD REDSTONE SCIENTIFIC INFO CTR ATTN: DOCUMENTS SECT, BLDG. 4484 REDSTONE ARSENAL, AL 35898	2
COMMANDER US ARMY FGN SCIENCE & TECH CTR ATTN: DRXST-SD 220 7TH STREET, N.E. CHARLOTTESVILLE, VA 22901	1

NOTE: PLEASE NOTIFY COMMANDER, ARMAMENT RESEARCH AND DEVELOPMENT CENTER, US ARMY AMCCOM, ATTN: BENET WEAPONS LABORATORY, SMCAR-LCB-TL, WATERVLIET, NY 12189, OF ANY ADDRESS CHANGES.

TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST (CONT'D)

	NO. OF COPIES
COMMANDER US ARMY MATERIALS & MECHANICS RESEARCH CENTER ATTN: TECH LIB - DRXMR-PL WATERTOWN, MA 01272	2
COMMANDER US ARMY RESEARCH OFFICE ATTN: CHIEF, IPO P.O. BOX 12211 RESEARCH TRIANGLE PARK, NC 27709	1
COMMANDER US ARMY HARRY DIAMOND LAB ATTN: TECH LIB 2800 POWDER MILL ROAD ADELPHIA, MD 20783	1
COMMANDER NAVAL SURFACE WEAPONS CTR ATTN: TECHNICAL LIBRARY CODE X212 DAHLGREN, VA 22448	1
DIRECTOR US NAVAL RESEARCH LAB ATTN: DIR, MECH DIV CODE 26-27, (DOC LIB) WASHINGTON, D.C. 20375	1 1
COMMANDER AIR FORCE ARMAMENT LABORATORY ATTN: AFATL/DLJ AFATL/DLJG EGLIN AFB, FL 32542	1 1
METALS & CERAMICS INFO CTR BATTELLE COLUMBUS LAB 505 KING AVENUE COLUMBUS, OH 43201	1

NOTE: PLEASE NOTIFY COMMANDER, ARMAMENT RESEARCH AND DEVELOPMENT CENTER, US ARMY AMCCOM, ATTN: BENET WEAPONS LABORATORY, SMCAR-LCB-TL, WATERVLIET, NY 12189, OF ANY ADDRESS CHANGES.

END

FILMED

11-85

DTIC